

Latest trends in the study of accretion and outflows around compact objects

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Abstract. Study of astrophysics of black holes and neutron stars has taken a new turn in the present decade with the realization that sub-Keplerian flows and the associated centrifugal barrier near the horizon or the surface of a neutron star play a major role in deciding the nature of the emitted spectra and the formation of outflows from the accreting matter. This region may remain steady or oscillate depending on the accretion rate, specific angular momentum and specific energy of the flow. Intricacies of oscillation may depend on the degree of feedback the inflow receives from the outflow. This region may emit hard or soft X-rays depending on relative numbers of hot electrons and soft photons intercepted by this region. We discuss how these properties come about and how they explain the observational results of black hole candidates.

Keywords: : Black holes, accretion, outflows, X-ray spectra

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1. Introduction

Several reviews on accretion processes on black holes and neutron stars have been written recently [1-3] where it is discussed that sub-Keplerian flows play major role in explaining hard/soft state transition, quasi-periodic oscillation of X-Rays and outflows from accretion around black holes. In the present review, we shall concentrate on observational results and how they could be explained with the latest solutions of accretion flows.

2. Generic properties of the advective accretion flows

Since the emergence of the standard Keplerian disk model [4] in seventies, purely rotating thick accretion disk models [5] were built in the eighties, with

nearly constant specific angular momentum λ , or where the angular momentum varies with radial distance in the form of a power-law [6]. These are the predecessors of current advective disk models [1-3], but the current models self-consistently include radial motions as well. Certain old models did attempt to include radial motion in the past [e.g., 7-6], but correct global solutions were not found.

An accreting matter must have a significant radial motion, especially close to the compact object. However, while on the horizon of a black hole, the velocity of matter attains the velocity of light, the matter must slow down and eventually stop on the hard surface of a neutron star. This behaviour is completely independent of the outer boundary condition. For any generic equation of state, this means that matter must be supersonic on the horizon and subsonic on a neutron star surface. A supersonic flow must be sub-Keplerian, and therefore the black hole accretion must deviate from a standard Keplerian disk [4]. Second, since infall time is very short compared with the viscous time to transport angular momentum, the specific angular momentum (λ) must be almost constant. This means that the centrifugal force λ^2/r^3 grows rapidly compared to the gravitational force $1/r^2$ and slows down the matter forcing it to be subsonic. Due to general relativistic effects (where gravity is really stronger than $1/r^2$ very close to the horizon), matter recovers from this quasi-stagnant condition, passes through another sonic point and finally enters into the black hole supersonically.

This temporary stagnation of matter due to centrifugal force (at the so-called centrifugal force driven boundary layer of a black hole, or CENBOL), especially true when viscosity is low [9], at a few to a few tens of Schwarzschild radii is of extreme importance in black hole physics. Slowed down matter would be hotter and would emit harder radiations. If viscosity is high, the Keplerian disk extends (from outside) to distances very close to the black hole, perhaps as close as the marginally stable orbit ($3R_g$, R_g being the Schwarzschild radius.). If viscosity is low, the inner edge may recede outwards to a few tens to a few hundreds of Schwarzschild radii. This is because angular momentum must be transported outward by viscosity and smaller viscosity transports slowly taking longer path length to reach a Keplerian disk. When the viscosity is so high that the specific angular momentum rapidly varies with distance, the centrifugal barrier weakens and the stagnation region disappears [10] since the sub-Keplerian region is confined roughly between $3R_g$ and R_g only.

When the stagnation region forms *abruptly* in a fast moving sub-Keplerian flow, a shock is said to have formed. Depending on the parameter space [11, 2-3], this shock may or may not be standing at a given radius. It may form and propagate to infinity [12], or may just oscillate [13-15], or may stand still [16-17]. In any case, generally shock forms and it decidedly affects the nature of the spectra manifesting itself through propagating noise, or quasi-periodic oscillation [13-15], or steady state spectrum [18-19], or the formation of the

quiescence state [20].

For a neutron star, the entire accretion may be sub-sonic and deviation from a Keplerian disk is not essential except in the narrow boundary layer where the rotational motion of the accreting matter must adjust to the rotational motion of the star. This is true when magnetic field is absent. In presence of a strong magnetic field, matter is stopped way ahead of the stellar surface and is bent back along the field line. If the flow deviates from a Keplerian disk, the ram pressure ρv_r^2 is higher and some matter can penetrate radially through the field line and directly hit the surface when the field is weaker. If this flow does become supersonic at any point, it must become subsonic just before hitting the surface [21].

Formation of a shock or the stagnation region (boundary layer) is generic in neutron star accretions. This is because the flow must stop on the surface and a boundary layer must be produced. However, in many cases, two shocks or stagnation regions may form. Chakrabarti [11] showed (see also, [9]) that for a neutron star, one shock is possible at a large distance (say, $r \gtrsim 10-15R_g$, where $R_g = 2GM/c^2$ is the Schwarzschild radius, G , M and c are the gravitational constant, mass of the black hole and the velocity of light respectively.) as in a black hole and another shock is possible at around $2.3 - 2.4R_g$. Oscillations of the outer shock would produce $1 - 10\text{Hz}$ and the oscillation of the shock at the inner shock would produce $\sim\text{KHz}$ oscillations. However, as Chakrabarti [9, 11] pointed out, inner shock would be possible only if the star is compact enough, otherwise, it would a normal boundary layer (as in a white dwarf [22]), which could also oscillate if the cooling time matches with the infall time [13], or, if the input parameters are such that a steady solution is not possible as in a black hole accretion [14].

In the centrifugal barrier dominated boundary layer of black holes and neutron stars, winds may form [2-3]. However, this phenomenon is more significant for a black hole than for a neutron star. Neutron stars are known to have magnetic fields and therefore most of the matter is stopped by it (unless the field is so weak that sub-Keplerian matter with larger ram-pressure penetrates it easily and hits the surface directly). Matter moves along the field lines to the polar region and most of the winds come out from that region only. On the contrary, black holes may not anchor magnetic fields. Most of the outflows can thus form at the CENBOL itself, due to its high outward pressure gradient force, and due to the high temperature. However, it is unlikely that purely hydrodynamic acceleration could cause a 'super-luminal' jet to form [23]. Since in the low-luminosity hard state CENBOL is hotter, the ratio $R_{in} = \frac{\dot{M}_{out}}{\dot{M}_{in}}$ should be higher in hard states than in soft-states (when the CENBOL is cooler) [20].

There is another region of outflow: at the boundary between the Keplerian and the sub-Keplerian matter. As Chakrabarti et al. [24-25] showed through extensive numerical simulation, angular momentum distribution at this transi-

tion region is super-Keplerian and therefore, outward centrifugal force is very strong. Thus, a good deal of centrifugally driven outflow at this region cannot be ruled out. In case of neutron stars, because of the same reason, outflows are possible not only in polar caps, but at the transition radius as well. If the magnetic field is non-aligned with the rotational axis, the Coriolis force on both sides of the disk would be slightly different and the QPO frequencies would split. Such behaviours have been observed in several neutron stars [26] and these essentially verify outflow solutions from transition regions [24-25].

3. Spectral properties from compact objects: expectations vs. observations

3.1. Steady state properties

If one denotes photon flux to be of the form, $\Phi_\nu \propto \nu^{-\alpha+1}$, (where ν is the photon frequency and α is the spectral index) then the energy spectrum would be $F_\nu = \int \Phi(\nu) d\nu \propto \nu^{-\alpha}$. The power is measured by $P_\nu = \int F_\nu d\nu \propto \nu^{-\alpha-1}$. When the power is larger in the hard X-ray region, the black hole is said to be in a hard state, whereas, when the power is larger in the soft X-ray region, the black hole is said to be in a hard state. Several black hole candidates are known to switch between hard and soft states [27-29].

In Fig. 1 (taken from [30], see also [9], [18]), a series of global solutions of the hydrodynamic equation with different viscosities are shown. The ratio λ/λ_K (Here, λ and λ_K are the specific angular momenta of the disk and the Keplerian angular momentum respectively.) is plotted as a function of the logarithmic radial distance. The coefficients of viscosity parameter are marked on the curves (see, [30] for other parameters). At $x = x_K$, the ratio $\lambda/\lambda_K = 1$ and therefore x_K represents the transition region where the flow deviates from a Keplerian disk. First, note that when other parameters (basically, specific angular momentum and the location of the inner sonic point) remain roughly the same, x_K changes inversely with viscosity parameter α_Π [9] (Only exception is the curve marked with 0.01. This is because it is drawn for $\gamma = 5/3$; all other curves are for $\gamma = 4/3$). If one assumes [9, 18], that alpha viscosity parameter *decreases* with vertical height, then it is clear from the general behaviour of Fig. 1 above that x_K would go up with height from the equatorial plane. The disk will then look like a sandwich with higher viscosity Keplerian matter flowing along the equatorial plane. Soft photons coming out of the Keplerian component are intercepted by the sub-Keplerian region and are up-scattered (inverse Comptonization) to produce power-law hard X-rays. When the Keplerian disk rate is high, there are enough soft photons to cool down the CENBOL completely and only soft X-rays are seen (except for a very weak power-law hard X-ray component extending upto a few hundred KeV about which we shall talk later). When the Keplerian rate is very small compared to the sub-Keplerian rate, the CENBOL cannot be cooled down, and spectrum

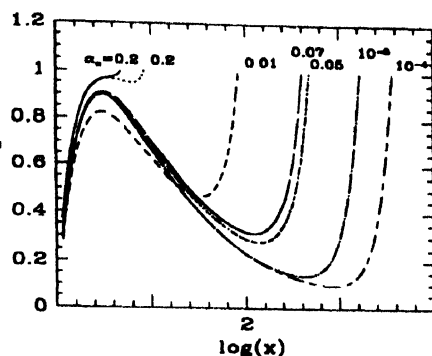


Figure 1. : Variation of λ/λ_K with logarithmic radial distance for a few solutions. Viscosity parameter α_{II} is marked on each curve. Note that except for the dashed curve marked 0.01 (which is for $\gamma = 5/3$, and the rest are for $\gamma = 4/3$), x_K , where $\lambda/\lambda_K = 1$, generally rises with decreasing α_{II} . Thus, low viscosity flows must deviate from the Keplerian disk farther out of the black hole [30].

is dominated by hard X-rays only. The Comptonization process which is responsible is 'thermal' Comptonization, since the energy received by photons are coming from the thermal motion of the hot electrons.

Independent of the outer boundary conditions (accretion rate, viscosity etc.), since matter must rush towards the black hole with a high velocity (after crossing the inner sonic point), a new phenomenon is observed as first noticed by Chakrabarti & Titarchuk [18]. This is known as the bulk motion Comptonization. Radiation is up-scattered as it hits radially moving electron, independent of whether the electrons themselves are hot or cold. In the hard state, this effect of up-scattering due to bulk motion is washed out by the up-scattering due to thermal motion. But in the soft state, when the electrons are cooled down to a few KeVs [18], this particular kind of up-scattering produces a power-law tail extending to a few hundred KeV. This is the true signature of a black hole accretion. A neutron star accretion would not have the bulk motion Comptonization and the power-law tail is not observed either. Several black holes have now been identified using this technique [31-32]. This method is clearly useful when one considers massive and supermassive black holes [33].

As the viscosity changes at the outer edge, the sub-Keplerian and Keplerian flows redistribute [12] and the inner edge of the Keplerian component also recedes or advances. When the Keplerian component advances (high viscosity, high rate), the soft state is achieved. When the Keplerian component recedes, the hard state is achieved. This behaviour is seen in many black hole candidates [32, 34].

On the other hand, to change states (hard to soft and vice versa), it is

sufficient if the sub-Keplerian component alone changes [18]. Yadav et al. [35] pointed out that the state change takes place in GRS1915+105 in matter of a few tens of seconds during quasi periodic oscillations. Since the viscous time-scale is much longer, the inner-edge of the Keplerian flow does not have time to move in or out within these short periods. Hence, appearance and disappearance of sub-Keplerian flows must be responsible. The exact mechanism will be discussed in detail later.

3.2. Time dependent properties

After years of X-ray bursts, an X-ray novae can become very faint and hardly detectable in X-rays. This is called the quiescent state. This property is built into the advective disk models. As already demonstrated (Fig. 1) x_K recedes from the black holes as viscosity is decreased. With the decrease of viscosity, less matter goes to the Keplerian component i.e., \dot{m}_d goes down. Since the inner edge of the Keplerian disk does not go all the way to the last stable orbit, optical radiation is weaker in comparison with what it would have been predicted by a standard disk model [4]. This behaviour is seen in V404 Cyg [36] and A0620-00 [37]. The deviated component from the Keplerian disk almost resembles a constant energy thick ion torus of earlier days [38].

In some large region of the parameter space the solutions of the governing equations are inherently time-dependent [1-3]. Just as a pendulum inherently oscillates while searching for a static solution, the physical quantities of the advective disks also show oscillations of the CENBOL region for some range in parameter space. This oscillation is triggered by competitions among various time scales (such as infall time scale, cooling time scales by different processes, including matter loss through outflows). Thus, even if black holes do not have hard surfaces, quasi-periodic oscillations could be produced. Although any number of physical processes such as acoustic oscillations [39], disko-seismology [40], or trapped oscillations [41] could produce such oscillation frequencies, modulation of 10 – 100 per cent or above cannot be achieved without bringing in the dynamical participation of the hard X-ray emitting region, namely, the CENBOL. By expanding back and forth (and puffing up and collapsing, alternatively) CENBOL intercepts variable amount of soft photons and reprocesses them. A typical observation is presented by Rao [42, see also, 43-44]. These observations can be readily explained by combination of cooling processes in CENBOL.

Imagine that an otherwise standing shock wave is perturbed to move outward against the direction of the inflow. The post-shock flow would be hotter than the steady-state value, and the cooling rate would be higher. The increased cooling would reduce the post-shock pressure which will cause collapse of the shock towards the black hole. In this case, the post-shock temperature would be lower, and the cooling will slow down. Ram pressure and thermal

pressure of the incoming flow would push the shock inward where the post shock-pressure is higher. This causes overshooting of the shock location and the shock starts moving outwards. This oscillation is quasi-sinusoidal in presence of a single cooling process, such as bremsstrahlung and Comptonization [13], but could be more complex looking if a large number of cooling processes are included. A outflow from CENBOL is also a kind of dynamical ‘cooling’ and the presence of an outflow causes evolution of the pattern of the oscillation such as shown in [42-44] for the galactic black hole GRS 1915+105. Details of the effects of the outflow may be in order in this context.

3.3. Effects of outflows on quasi-periodic oscillations

Chakrabarti & Bhaskaran [45] first pointed out that outflows are more easily produced from a ‘sub-Keplerian’ flow, *even if* they are centrifugal pressure driven. A simple computation of the outflow from CENBOL area based on quasi-spherical accretion yields [2-3, 46] the ratio of outflow to inflow rate to be:

$$\frac{\dot{M}_{out}}{\dot{M}_{in}} = R_{\dot{m}} = \frac{\Theta_{out}}{\Theta_{in}} \frac{R}{4} [f_0]^{3/2} \exp\left(\frac{3}{2} - f_0\right) \quad (1)$$

where, R is the compression ratio of the flow at the CENBOL and Θ_{in} and Θ_{out} are the solid angles of the inflow and the outflow respectively. $f_0 = R^2/R - 1$. Location of the sonic point of the outflow is at $r_c = f_0 r_s$ [3, 46].

Figure 2 shows the ratio $R_{\dot{m}}$ as a function of the compression ratio R (plotted from 1 to 7) assuming $\Theta_{out} \sim \Theta_{in}$ for simplicity. Note that if the compression (over and above the geometric compression) does not take place (namely, if the CENBOL does not form), then there is no outflow in this model. Indeed, for $R = 1$, the ratio $R_{\dot{m}}$ is zero! Thus the driving force of the outflow is primarily coming from the hot, compressed region. This picture is qualitatively supported by more detailed analysis [20].

One can discuss the effects of outflows [46] of a stellar black candidate such as GRS 1915+105 which exhibits QPOs as well as periodic changes to burst and quiescences state in a matter of tens of seconds. Particularly interesting is that it shows QPO of around $\nu_I = 1 - 10\text{Hz}$ and sometimes bursts at around $\nu_H = 0.01\text{ Hz}$ [43]. It also seems to show a QPO at $\nu_H \sim 67\text{Hz}$. It has been pointed out before that the spectrum of CENBOL may be softer in presence of outflows [47]. This is because CENBOL would have lesser matter (for a given soft photon flux). If the behaviour of outflow rate truly depends on compression ratio as in Fig. 2, one may be able to explain many complex properties, though details have to be worked out. An interesting property of shock-compressed $R_{\dot{m}}$ (Fig. 2) is that the outflow rate is peaked at around $R = 2.5$, i.e., when the shock is of ‘average’ strength. On either side, the outflow rate falls off very rapidly. As $R_{\dot{m}}$ [Eq. (1)] is independent of shock location r_s , shocks oscillating with time period similar to the infall time in $r < r_s$ region (causing a 1-10Hz

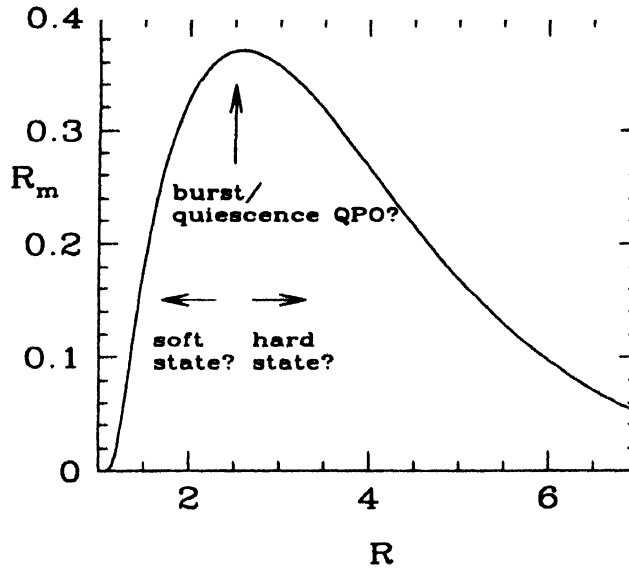


Figure 2. : Ratio \dot{R}_m of the outflow rate and the inflow rate as a function of the compression ratio of the gas at the dense region boundary. Solid angles subtended by the inflow and the outflow are assumed to be comparable for simplicity. Note that the ratio peaks at intermediate compression $R \sim 2.5$. Burst/quiescence feature of transient sources are related to this behaviour of the outflow rate.

QPO in the hard state [13, 18-19]) will continue to have outflows and gradually fill the (sonic) sphere of radius $r = r_c = f_0 r_s / 2$ ($f_0 = R^2 / R - 1$) (where matter is slowly moving) till $\langle \rho \rangle r_c \sigma_T \sim 1$ when this region would be cooled down catastrophically by inverse Comptonization. At this stage: (a) $r < r_c$ region would be drained to the hole in $t_{fall} \sim r_c^{3/2} 2GM/c^2 = 0.1 (r_s/50)^{3/2} (M/10M_\odot) (f_0/4)$ seconds. This is typically what is observed for GRS1915+105. [35]. If on the other hand, the outflow becomes locally supersonic (due to sudden cooling) the flow would run away outward forming a shock in the outflow. (b) In a cooler accreting gas, the shock will disappear, and a smaller compression ratio ($R \rightarrow 1$) would 'cut off' the outflow (Fig. 2). In other words, during burst/quiescence QPO phase, the outflow would be blobby. (c) The black hole will go to a soft state during a short period. If the angular momentum is large enough, this brief period of soft state may be prolonged to a longer period of tens of seconds depending on how the centrifugal barrier is removed by viscosity (generated during shock oscillations). There are actually two possibilities, if the sonic sphere is cooled down, the outflow could become locally supersonic, and the burst state becomes momentary. If on the other hand, the flow in

sonic sphere remains subsonic even after cooling, and returns back to the black hole due to lack of driving force, then the burst state may be long-lived when angular momentum is high, otherwise short-lived as discussed above. It is also possible that cooler outflow takes a longer time to be drained out, and caused prolonged burst state. A signature of bulk motion Comptonization in the burst-state spectrum would ensure the drainage by the black hole. Since $f_0 \sim 4$, the sonic sphere is typically four times bigger and would intercept four times larger number of photons. Thus the count rate should fluctuate roughly by a factor of f_0 in between quiescence and burst states. This is precisely what is observed [48].

There is one good test to check if the QPOs are actually from shock oscillations. When CENBOL oscillates, the fractional change in hard X-ray emission is very high compared to the fractional change in softer X-rays (since most of the contribution to soft X-rays is from the Keplerian disk whose properties are not modulated with QPOs). This is clearly demonstrated in this object (Fig. 5 of [48]). Similarly, when the Keplerian accretion rate is increased, the cooling time goes down, and one would expect the frequency to increase with soft luminosity. In soft states QPOs are usually absent, but in burst phase if the QPO is present it is seen in higher frequency only (Fig. 2 of [48]).

Typically, a shock located at r_s (measured hereafter in units of $R_g = 2GM/c^2$), produces an oscillation of frequency [13],

$$\nu_I = \frac{1}{t_{ff}} \sim \frac{1}{R} r_s^{-3/2} \frac{c}{R_g} \quad (2)$$

where, R is the compression ratio of the gas at the shock which the infall velocity goes down and therefore the infall time t_{ff} goes up. For a gas of $\gamma = 4/3$, $R \sim 7$ and for $\gamma = 5/3$, $R \sim 4$ respectively when the shock is strong. Thus, for instance, for a $\nu_I = 6\text{Hz}$, $r_s \sim 38$ for $M = 10M_\odot$ and $\gamma = 4/3$. For $\nu_H = 67\text{Hz}$, $r_s \sim 8$ for the same parameters. Chakrabarti & Titarchuk [18] pointed out that since black hole QPOs show a large amount of photon flux variation, they can not be explained simply by assuming some inhomogeneities, or perturbations in the flow.

Since the sonic point location is a function of the shock location, one would expect that by time of filling the sonic sphere (and time to cool it, i.e., the duration of the quiescence state) would be correlated with the QPO frequency. The volume filling time of the sonic sphere is [46],

$$t_{fill} = \frac{4\pi r_c^3 \langle \rho \rangle}{3\dot{M}_{out}} \quad (3)$$

The cooling starts when $\langle \rho \rangle r_c \sigma_T \gtrsim 1$, $\sigma_T = 0.4$ is the Thomson scattering cross-section. Thus the duration of the quiescence state is given by,

$$t_q = \frac{4\pi r_c^2}{3\dot{M}_{out} \sigma_T} \quad (4)$$

Because of uncertainties in Θ_{in} , Θ_{out} and \dot{M}_{in} (see, eq. 1) (subscript 'in' refers to the inflow rate) we define a new parameter,

$$\Theta_{\dot{m}} = \frac{\Theta_{out}}{\Theta_{in}} \frac{\dot{M}_{in}}{\dot{M}_{Edd}}. \quad (5)$$

Using these equations, we get the expression for t_q as,

$$t_q = \frac{41.88}{(R-1)^{1/2}} \frac{r_s^2 R_g^2 \exp(f_0 - \frac{3}{2})}{\dot{M}_{Edd} \Theta_{\dot{m}}} \text{ s}. \quad (6)$$

(where we brought back R_g factor to get cgs unit.) Or, eliminating shock location using eq. (2), we obtain,

$$t_q = 56.4 \frac{\exp(f_0 - \frac{3}{2})}{R^{4/3} (R-1)^{1/2} \Theta_{\dot{m}}} \left(\frac{M}{10M_{\odot}} \right)^{-1/3} \nu_I^{-4/3} \text{ s} \quad (7)$$

For an average shock, $2.5 < R < 3.$, the result is insensitive to the compression ratio. Using average value of $R = 2.9$ and for $\Theta_{\dot{m}} \sim 0.4$ (which corresponds to 0.4 Eddington rate for $\Theta_{out} \sim \Theta_{in}$) we get,

$$t_q = 461.5 \left(\frac{0.4}{\Theta_{\dot{m}}} \right) \left(\frac{M}{10M_{\odot}} \right)^{-1/3} \nu_I^{-4/3} \text{ s} \quad (8)$$

Thus, the duration of the off-state must go down rapidly as the QPO frequency increases if the net accretion remains fixed. Similarly, if the hot post-shock gas of height $\sim r_s$ intercepts n soft photons per second, from the pre-shock Keplerian component [18], it should intercept about $f_0 n$ soft photons per second when the sonic sphere of size r_c is filled in. Thus, the photon flux in the burst phase should be about $f_0 = R^2(R-1)^{-1} \gtrsim 4$ times larger compared to the photon flux in quiescence state. Depending on the degree of flaring, the interception may be much higher.

In Fig. 3 we show results of three observations of X-ray transient GRS 1915+105 (see, [48] for the observational results) where we plot ν_I along y-axis and t_q along x-axis. The dotted curve shows eq. (8) which passes close to June 18th, 1997 data and Oct 7th, 1996. A short dashed curve is drawn using the same equation but assuming a law ν^{-2} and for accretion parameter $\Theta_{\dot{m}} = 0.15$. Several reasons, such as slower infall velocity in presence of angular momentum and/or assumption constant volume filling rate $\dot{M}_{out}/<\rho>$ would modify the eq. (8) to have an inverse square dependence on ν . However exact derivation with these new physical inputs would require more parameters such as angular momentum or mean density. The result of May 26th, 1997 falls on a different inverse square curve (long dashed) valid for $\Theta_{\dot{m}} = 0.055$ indicating a change in outflow geometry and/or a change in accretion rate.

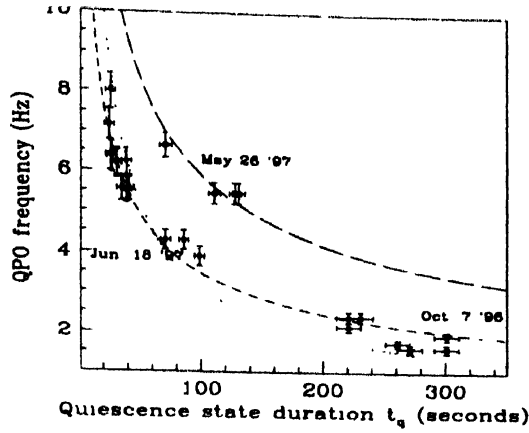


Figure 3. : Variation of QPO frequency ν_I with duration of quiescence states t_q . Dotted curve is the $t_q \propto \nu^{-4/3}$ law (eq. 8) derived using simple free-fall velocity assumption. Dashed curves are $t_q \propto \nu^{-2}$ fits suggesting modified velocity law. The general agreement points to the shock oscillation model

4. Future directions

Presence of a sub-Keplerian disk (hot or cold) close to a black hole has forced people to re-analyze some of the old problems and several new findings have been made. For instance, recently it has been noted that significant nucleosynthesis inside this region can destabilize a disk and cause sonic point oscillations [30]. However, an old problem of in-spiraling and coalescence of two point masses have re-surfaced. It is recognized that the presence of a sub-Keplerian disk [49] can seriously change the nature of gravitational wave emission if the accretion rate is high enough. Because of its importance in next generation gravitational wave astronomy, it is worth repeating some of the arguments.

Chakrabarti [50] first pointed out that the accretion disks close to the black hole need not be Keplerian and it would affect the gravitational wave properties. The radiation pressure dominated disks are likely to be super-Keplerian in some range which would transfer angular momentum to the orbiting companion and in some extreme situations, can even stabilize its orbit from coalescing any further. This was later verified by time-dependent numerical simulations [51]. Fig. 1 shows, however, that in most of the regions flow likes to remain sub-Keplerian ($\lambda/\lambda_K < 1$) rather than super-Keplerian ($\lambda/\lambda_K > 1$). Assume that a companion of mass M_2 is in an instantaneous circular Keplerian orbit around a central black hole of mass M_1 . This assumption is justified, specially when the orbital radius is larger than a few Schwarzschild radius where the

energy loss per orbit is negligible compared to the binding energy of the orbit. The rate of loss of energy dE/dt in this binary system with orbital period P (in hours) is given by (see, [49] and references therein),

$$\frac{dE}{dt} = 3 \times 10^{33} \left(\frac{\mu}{M_{\odot}} \right)^2 \left(\frac{M_{tot}}{M_{\odot}} \right)^{4/3} \left(\frac{P}{1hr} \right)^{-10/3} \text{ergs sec}^{-1}, \quad (4)$$

where,

$$\mu = \frac{M_1 M_2}{M_1 + M_2}$$

and

$$M_{tot} = M_1 + M_2.$$

The orbital angular momentum loss rate would be,

$$R_{gw} = \frac{dL}{dt}|_{gw} = \frac{1}{\Omega} \frac{dE}{dt}, \quad (5)$$

where, $\Omega = \sqrt{GM_1/r^3}$ is the Keplerian angular velocity of the secondary black hole with mean orbiting radius r . The subscript 'gw' signifies that the rate is due to gravitational wave emission. In presence of an accretion disk co-planer with the orbiting companion, matter from the disk (with local specific angular momentum $\lambda(r)$) will be accreted onto the companion at a rate close to its Bondi accretion rate [49],

$$\dot{M}_2 = \frac{4\pi \bar{\Lambda} \rho (GM_2)^2}{(v_{rel}^2 + a^2)^{3/2}} \quad (6)$$

where, ρ is the density of disk matter, $\bar{\Lambda}$ is a constant of order unity (which we choose to be 1/2 for the rest of the paper), $v_{rel} = v_{disk} - v_{Kep}$ is the relative velocity of matter between the disk and the orbiting companion. The rate at which angular momentum of the companion will be changed due to Bondi accretion will be,

$$R_{disk} = \frac{dL}{dt}|_{disk} = \dot{M}_2 (\lambda_K(r) - \lambda(r)). \quad (7)$$

Here, λ_K and λ are the local Keplerian and disk angular momenta respectively. The subscript in the left hand side signifies the effect is due to the disk. If some region of the disk is sub-Keplerian ($\lambda < \lambda_K$), the effect of the disk would be to reduce the angular momentum of the companion further and hasten coalescence. If some region of the disk is super-Keplerian, the companion will gain angular momentum due to accretion, and the coalescence is slowed down.

In order to appreciate the effect due to intervention of the disk, we consider a special case where, $M_2 \ll M_1$, $\lambda \ll \lambda_K$ and $v \ll a$ (subsonic flow). In this case, $\mu \sim M_2$ and $M_{tot} \sim M_1$. The ratio R_{d-g} of these two rates is,

$$R_{d-g} = \frac{R_{disk}}{R_{gw}} = 1.518 \times 10^{-7} \frac{\rho_{10}}{x^{3/2}} x^4 M_8^2 \quad (8)$$

Here, x is the companion orbit radius in units of the Schwarzschild radius of the primary, M_8 is in units of $10^8 M_\odot$, ρ_{10} is the density in units of $10^{-10} \text{ gm cm}^{-3}$ and T_{10} is the temperature of the disk in units of 10^{10} K . It is clear that, for instance, at $x = 10$, and $M_8 = 10$, the ratio $R \sim 0.15$ suggesting the effect of the disk could be a significant correction term to the general relativistic loss of angular momentum. The ratio R_{d-g} is independent of the mass of the companion black hole, as long as $M_2 \ll M_1$. If the flow is highly supersonic ($v \gg a$) then the result is independent of temperature of the flow.

Chakrabarti [49] shows that the effect is especially important when the net accretion rate (Keplerian and sub-Keplerian) is high enough. (Effects shown in [49] were computed for $\dot{M} = 1000 \dot{M}_{Edd}$, for instance. Clearly, the net effect goes down with accretion rate.) In a hard state, although the Keplerian rate is low, the sub-Keplerian rate could be high and this effect is important. However, in the soft state, the disk remains basically Keplerian except last two-three Schwarzschild radii. Thus, the effect would be negligible except in this region. A combination of results from electromagnetic spectrum and gravitational wave spectrum would be more effective to put limits on the parameters such as the masses of the black holes, individual accretion rates and separation between the black holes etc. Clearly this is going to be an exciting task.

An effect so far ignored in this context is the formation of outflows from the CENBOL region [2, 3, 20, 52] which may 'mess-up' computation even farther. It has been shown [20, 51] that profuse outflows, to the extent causing evacuation of the disk can be produced. In this case, the effect would clearly be negligible, but the non-linearity of the outflow rate with inflow rate (Fig. 2) makes the effect more difficult to incorporate.

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